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Note

When a Three-Way Fabric Hangs Together

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A simple condition is given that determines when a three-way fabric hangs together. © 1985 Academic Press, Inc.

Nowakowski and Skillicorn [1] investigate periodic fabrics formed by the intersection of three sets of parallel strands at 120° to each other and raise the question of when such a fabric hangs together.

Represent the three sets of strands by three sets X, Y, Z of vertices and draw a directed arc from vertex u to vertex v if the strand represented by u passes under the strand represented by v. The result is a tripartite tournament. Suppose that X has m vertices with scores $a_1,..., a_m$, that Y has n vertices with scores $b_1,..., b_n$ and that Z has p vertices with scores $c_1,..., c_p$ with each list in nondecreasing order.

The fabric does not hang together if and only if there is a nonempty proper subset of the set of strands which lifts off the remainder. This amounts to saying that the tripartite tournament is reducible. This is so if and only if there is a nonempty proper subset of the set of vertices whose scores add up to the total possible number of arcs between these vertices. Thus the fabric hangs together unless

$$\sum_{i=1}^{s} a_i + \sum_{j=1}^{t} b_j + \sum_{k=1}^{u} c_k = tu + us + st$$

for some values of s, t, u (the value zero being permitted but the two cases $s = t = u = 0$ and $s = m$, $t = n$, $u = p$ being unacceptable).

REFERENCE

1. R. NOWAKOWSKI AND D. SKILLICORN, Three-way weaving, *Congr. Numer.* **36** (1982), 153–159.

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